Transition Materials

GCSE Maths to A-Level Maths

Pack A

Contents:

- Indices
- Surds
- Rearranging Fractions
- Solving Linear Equations
- Sketching Linear Graphs
- More Brackets
- Further Factorising
- Rearranging Factorising
- Solving Quadratics

Produced by the Advanced Mathematics Support Program for the Arthur Terry School

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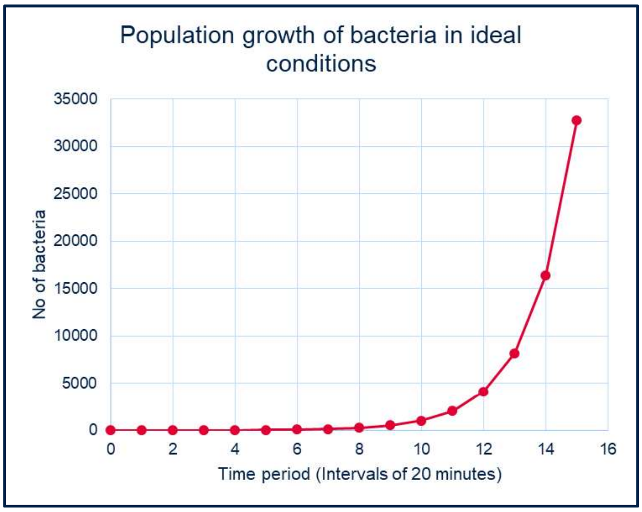
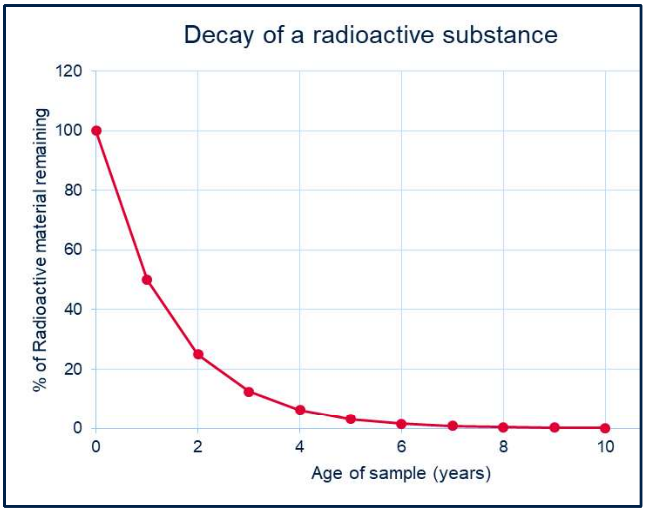
Indices

Did you know?

Indices are also referred to as exponents





e.g. $2^{3} = 8$

3 is the 'exponent'

$$2^3 = \underbrace{2 \times 2 \times 2}$$

It tells us how many times a number is multiplied by itself

This is where exponential graphs come from!

Indices 1

Simplify the following

1. $x^3 \times x^8 =$

2. $\frac{9^8}{9} =$

3. $(2^3)^5 =$

4. $\frac{4^4 \times 4}{(4^2)^3} =$

5. $16^{\frac{1}{2}} =$

6. What is the reciprocal of 16

7. What is 4^{-3}

8. What is $\left(\frac{2}{5}\right)^{-1}$

Indices 2

Simplify the following

1. $t^5 \times t^4 =$

2. $\frac{8^7}{8^2} =$

3. $(3^4)^2 =$

4. $\frac{5^7 \times 5}{(5^3)^3} =$

5. $(8)^{\frac{1}{3}} =$

6. $y^0 =$

7. What is $4^{-3} =$

8. What is $\left(\frac{2}{3}\right)^{-2} =$

Roots and Indices Maze

Can you find the way from one side of the table to the other?

- Begin in the highlighted box
- Move vertically or horizontally one box at a time ……..no diagonal moves allowed
- You may only land on boxes which are equivalent in value to the highlighted one

$2^6 \times 2^3$	$3^2 \times 2^3$	$(\sqrt{16})^2$	$(2^3)^3$	$8^3 \div 8$	$4^4 \times 4^{-3}$	$(\sqrt[3]{8})^4$	8×4^2
$\sqrt{8}^3$	$(2^3)^2$	$8^7 \times 8^{-5}$	4^3	$2^{-2} \times 2^7$	64^0	$2^5 \times 2^3$	$4^7 \div 2^3$
$(\sqrt{64})^3$	8^2	$2^2 \times 2^3$	$2^3 \times 2^3$	$(2^3)^3$	$(\sqrt[3]{8})^6$	$4^6 \times 4^{-3}$	$2^2 \times 4^2$
2^6	$(\sqrt{64})^2$	$4^6 \times 4^{-2}$	$(\sqrt{16})^3$	$(2^2)^4$	$8^3 \div 2^3$	$2^{-3} \times 2^7$	$(2^2)^4$
3^5	$2^6 \times 2^1$	8^3	$4^5 \div 2^4$	$(-4)^{-3}$	$(2^2)^3$	$(\sqrt{8})^3$	$4^6 \div 2^6$
$4^3 \times 4^{-3}$	$(2^5)^1$	$(\sqrt[3]{64})^2$	$2^3 \times 8$	$2^{-1} \times 2^7$	$(\frac{1}{4})^{-3}$	16^2	64

Hint : What is the value of 2^6

Matching Pairs

Match the expressions in Column A with their equivalent expression in Column B

A	B
$\left(\frac{9}{16}\right)^{\frac{1}{2}}$	$\frac{3}{2}$
$(4)^{\frac{3}{2}}$	8
$(-5)^{-2}$	$\frac{1}{16}$
$(16)^{-\frac{3}{2}}$	$\frac{1}{4}$
$(2)^{-3}$	$\frac{3}{4}$
$(64)^{-\frac{1}{3}}$	$\frac{1}{25}$
$\left(\frac{4}{9}\right)^{-\frac{1}{2}}$	$\frac{1}{8}$
4^{-2}	$\frac{1}{64}$

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Surds

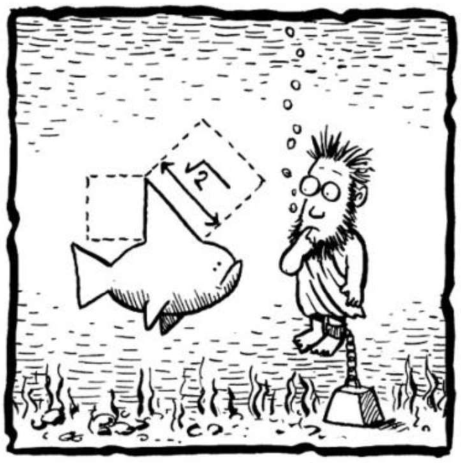
Did you know?

Maths can be murderous!

You will have heard of Pythagoras and his theorem but have you heard of Hippasus who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for $\sqrt{2}$. Following which, he was drowned at sea!



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Surds 1

1. Simplify $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

2. Simplify $\sqrt{2} \times \sqrt{6}$

3. Simplify fully $(4\sqrt{3})^2$

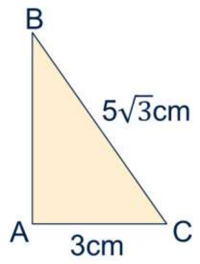
4. Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$

5. Calculate $\frac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of $\frac{4}{\sqrt{3}}$

7. A rectangle has an area of $8\sqrt{15}$ cm^2 and a length of $2\sqrt{3}$ cm. Find the width of the rectangle

8. Find the length AB



Surds 2

1. Simplify $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

2. Simplify $2\sqrt{b} \times 4\sqrt{3}$

3. Simplify fully $(4\sqrt{5})^2$

4. Write $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$ in the form $k\sqrt{3}$

5. Simplify $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of $\frac{2\sqrt{2}}{\sqrt{5}}$

7. Evaluate $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$

8. A triangle has base of $3\sqrt{2}$ and a perpendicular height of $5\sqrt{8}$

   Calculate the area of the triangle

Another Brick in the Wall

Complete the empty boxes in the pyramid.

Each box is the sum of the two boxes directly below it.

				$31\sqrt{3}+18\sqrt{5}$				
$\sqrt{75}$		$2\sqrt{3}$		$\sqrt{245}$		$-\sqrt{180}$		$2\sqrt{243}$

Hint: You may need to simplify some of the surds in the bottom row to get started.

True or False

Decide if each of the following expressions is *True* or *False*

1. $\sqrt{9}+\sqrt{4}=\sqrt{13}$
2. $\sqrt{a}\times\sqrt{b}=\sqrt{c}$
3. $\sqrt{(8)^2}=8$
4. $10\sqrt{2}=\sqrt{8}$
5. $\frac{\sqrt{12}\times\sqrt{3}}{\sqrt{9}}=2$
6. $\sqrt{2}^3=2\sqrt{2}$
7. $\sqrt{ab}^2=ab$
8. $2\sqrt{100}=\sqrt{200}$

Are there some statements that are ‘Sometimes true’ but not ‘Always true’? Explain why.

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Rearranging Fractions

Did you know?

Comparing Fractions

To order fractions you can compare the product of their diagonals

Compare these two fractions $\frac{5}{12}$ and $\frac{6}{13}$

$5 \times 13 = 65$ $\quad$ $6 \times 12 = 72$

$$\frac{5}{12} \quad \frac{6}{13}$$

as $72 > 65$ then $\frac{6}{13}$ is larger than $\frac{5}{12}$

Compare these two fractions $\frac{42}{98}$ and $\frac{12}{28}$

$42 \times 28 = 1176$ $\quad$ $98 \times 12 = 1176$

$$\frac{42}{98} \quad \frac{12}{28}$$

This means $\frac{42}{98} = \frac{12}{28}$ so are **equivalent** fractions

If fractions are equivalent then the product of their diagonals will always be equal!

How could you use this to help you when rearranging or solving equations involving fractions?

Fractions 1

1. Rewrite the formula to make time the subject

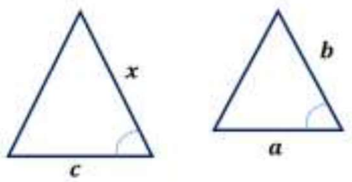
$$Speed = \frac{distance}{time}$$

2. Rearrange to make a the subject

$$\frac{x}{y} = \frac{a}{b}$$

3. Make x the subject of $tan\theta = \frac{y}{x}$

4. These triangles are similar.
Show that $x = \frac{cb}{a}$



5. Make x the subject of $x = \frac{h+k}{a}$

6. Make x the subject of $x + a = \frac{x+b}{c}$

7. Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

8. Make y the subject of $y\left(\sqrt{3} + \sqrt{2}\right) = x$
And write in the form $y = x(\sqrt{a} + \sqrt{b})$

Fractions 2

1. Make x the subject of $bc = \frac{x}{a}$

2. Make e the subject of $x = \frac{y}{e^2}$

3. Write a in terms of x, y, z and b.

$$\frac{b-xa}{z} = y$$

4. Make v the subject $C = \frac{v^2-ta}{x}$

5. Rearrange to make x the subject of

$$\frac{2}{x} + 5 = 6y$$

6. Make x the subject of

$$4F = F + \frac{a}{y+x}$$

7. Make y the subject of

$$\sqrt{\frac{m(y+a)}{y}} = y$$

8. A cylinder has a radius 3 cm and height h cm. The total surface area is $30x\ cm^2$
Find an expression for surface area and write h in terms of x and π

Wrong Steps

Each expression has been written in different ways

- Which are not correct rearrangements?
- Can you explain what's gone wrong?

	$c = \frac{3e^2}{d}$
A.	$d = 3e^2 - c$
B.	$cd = 3e^2$
C.	$\frac{d}{e^2} = \frac{c}{3}$
D.	$\frac{1}{3}c = \frac{e^2}{d}$
E.	$d = \frac{3e^2}{c}$

	$\frac{sinx}{4} = \frac{siny}{a}$
A.	$\frac{a}{4} = \frac{siny}{sinx}$
B.	$siny = \frac{4}{asinx}$
C.	$sinx = \frac{4siny}{a}$
D.	$asinx = 4siny$
E.	$a = \frac{sinx}{4siny}$

	$\frac{T-a}{T+a} = \frac{x}{y}$
A.	$x(T+a) = y(T-a)$
B.	$xy - ay = yT - ya$
C.	$a = \frac{y(T-a)}{x+y}$
D.	$xa + ya = yT - xT$
E.	$a = \frac{x+y}{yT-ya}$

	$a - \frac{b^2}{d} = ce$
A.	$b^2 = d(a+ce)$
B.	$a = ce + \frac{b^2}{d}$
C.	$\frac{b^2}{d} = a - ce$
D.	$\frac{b}{\sqrt{d}} = \sqrt{a} - \sqrt{ce}$
E.	$b = \pm\sqrt{d(a-ce)}$

	$y + b = \frac{ay+e}{b}$
A.	$by + b^2 = ay + e$
B.	$by - ay = e + b^2$
C.	$y = \frac{e-b^2}{b-a}$
D.	$e = b(y+b) - ay$
E.	$y(b-a) = \frac{e-b^2}{y}$

Prove it

Using your rearranging skills can you prove each of the following

If $\boldsymbol{a = \frac{b}{b+c}}$

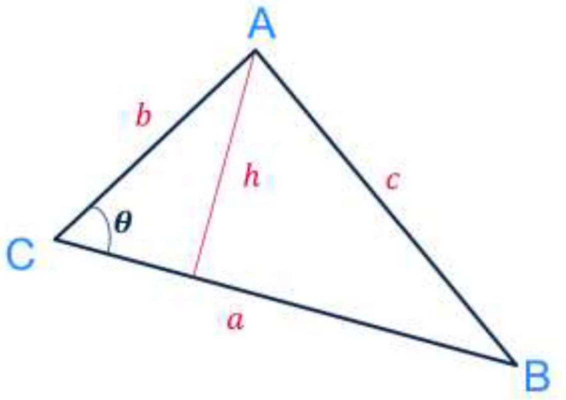
Show that $\boldsymbol{\frac{a}{1-a} = \frac{b}{c}}$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

Missing Steps

Complete the steps and fill in the blanks to find an expression for the area of triangle ABC



1. On the diagram draw a perpendicular line from A to BC

2. Label the perpendicular line, h h

3. Find an expression for the perpendicular height, h

$$h =$$

Hint: you might want to use some trigonometry here

4. Write down the expression for the base of the triangle

$$base =$$

5. Write down an expression to find the area of this triangle using your expressions for $base$ and $perpendicular\ height$

$$area =$$

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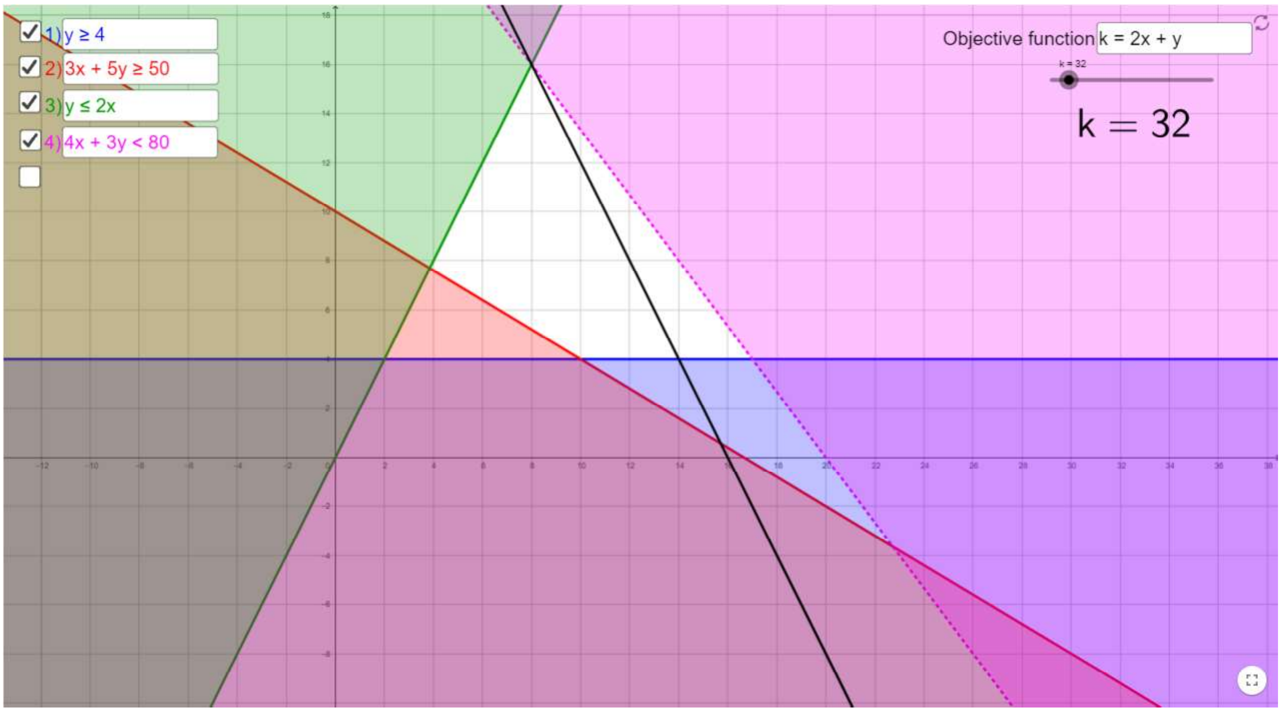
Solving Linear Equations

Did you know ?

Linear programming is a method that involves solving a set of linear equations or inequalities in order to find the best solution.

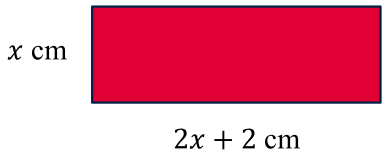


It is very useful in industry for finding the best level of production, or the maximum profit depending on varying costs, sales, mix of products or availability of labour etc…

Solving Linear 1

Solve the equations

1. $8x - 3 = 5x + 13$
2. $3x + 1 > 10$ and $2x + 7 < 15$
3. $3(x + 6) > 12$
4. $24 - 3x = 9$
5. $14 \geq 8 + 5x$
6. $6 - 2x < 5x + 34$
7. $\frac{2x+3}{7} = \frac{4x-5}{3}$
8. The perimeter of the rectangle is 24cm.
   Find the value of x.

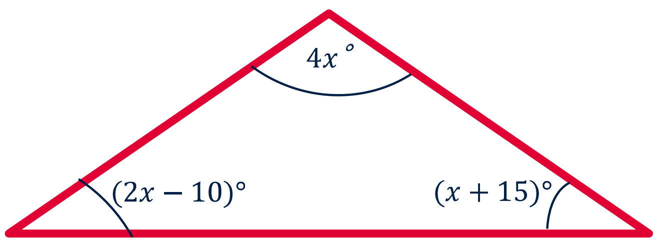


Solving Linear 2

Solve the equations

1. $6x + 5 = 47$
2. $5x + 7 = x + 25$
3. $7(x - 4) = 14$
4. $29 - 4x < 22$
5. $3x < 2x - 1 < 4x + 2$

   Hint: Split into two inequalities
6. $19 + 2x = 3x + 15$
7. $\frac{3x-1}{5} \geq \frac{3x+5}{2}$
8. Find the value of x in the triangle below



Piggy in the Middle

The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

5			23	

- What number goes in the right-hand cell?

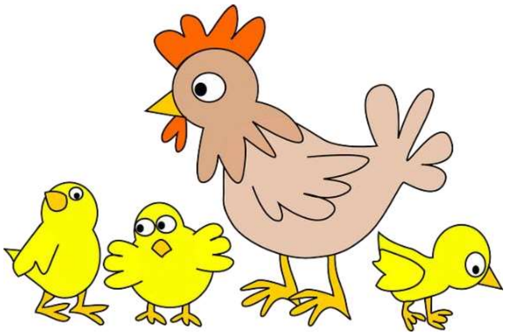
Chicken Run

Victoria has just bought some chickens. She wants to keep them safe in a small enclosure.

The enclosure will be a rectangle where the length is 3m longer than the width.

Victoria has only got 30m of fencing. The area of the enclosure has to be greater than 20m^2.
The length and width are integers.

- How many different size enclosures can Victoria make?



Crack the code

Can you decode this message?

12	1	4		7	5	3	12	4	2	5					
7	4	3	3	6	15	4		9	2	6	9	8	4	10	

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

1. $$\frac{4r}{d-4}+\frac{2h}{s}=2$$

2. $$\frac{g-9}{y+4}=\frac{2}{3}$$

3. $$3rh+m=13$$

4. $$\frac{4g}{5}=12$$

5. $$\frac{2c-5+3(c-2)}{2c-1}=2$$

6. $$e^3<72$$

7. $$\frac{s+3y}{8s}=\frac{3}{4}$$

8. $$\frac{6k}{s}-5=11$$

9. $$100<t^2<169$$

10. $$\frac{8}{3a}=\frac{4}{a+3}$$

11. $$\frac{6r+8}{y}=4$$

12. $$2(3m+4)=7m+1$$

Hint:

Try solving the equations in the following order: 4, 5, 10, 2, 11, 12, 7, 8, 3, 1, 9, 6

Linear Simultaneous Equations

There are two main ways to solve simultaneous equations.

Elimination

$$3x + 2y = 9$$

$$5x - 2y = -1$$

Add the two equations together to eliminate y

$$8x = 8$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$3 \times 1 + 2y = 9$$

$$3 + 2y = 9$$

$$2y = 6$$

$$y = 3$$

Substitution

$$y + 3x = 5$$

$$2y + 7x = 11$$

Rearrange the first equation in terms of y and then substitute into the second equation

$$2(5 - 3x) + 7x = 11$$

$$10 - 6x + 7x = 11$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$y + 3 \times 1 = 5$$

$$y + 3 = 5$$

$$y = 2$$

Which method is best and when?

Solve the following:

1.

$$2x + y = 7$$

$$2x - y = 1$$

2.

$$3x + 2y = 7$$

$$3x + 5y = 4$$

Which method will you use and why?

3.

$$y = 4x + 3$$

$$3x + 2y = 28$$

4.

$$4x + 3y = -4$$

$$6x - 2y = 7$$

Maths at the Movies

Maths movie makes millions!

"Our latest movie 'Sum-body loves you' has generated £15 million in online sales and rentals in the first week of it being released" Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.

Is there really no way of finding out how many rentals and how many sales there were of the film?

Use what you have learnt so far to calculate how many individual rentals and sales there were of 'Sum-body loves you'

Taxi!

There are two taxi companies

Initial Charge: £x then £1 per mile	Initial Charge: £$2x$ then 80p per mile

They both charge £12 for a journey of the same distance.

- What is the distance?
- What is the value of x?

Solving Graphically

Use the graphs to solve these pairs of equations

1. $3x + y = 10$

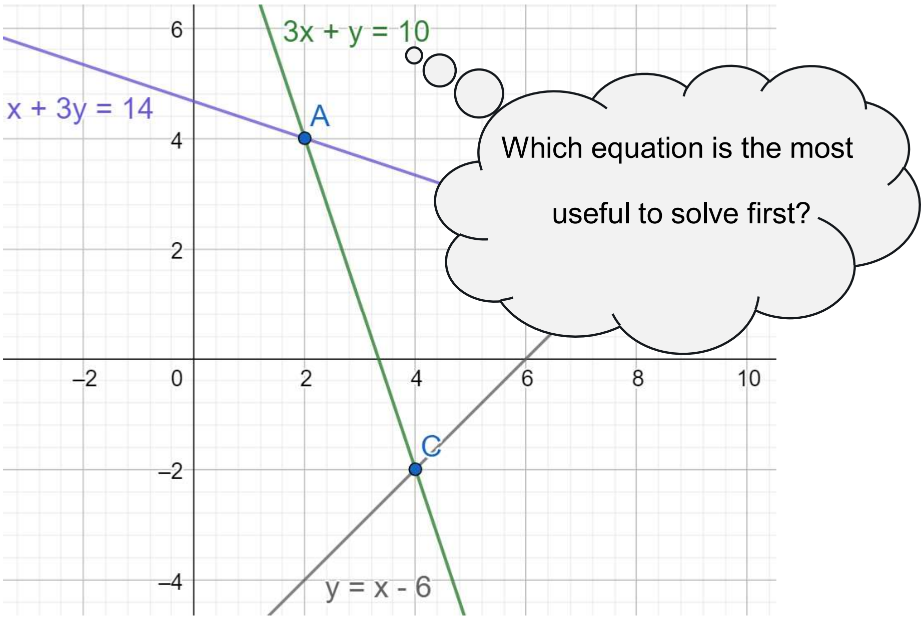
$x + 3y = 14$

2. $y = x - 6$

$3x + y = 10$

3. $x + 3y = 14$

$y = x - 6$



Puzzle to Ponder

Can you explain algebraically why there are no solutions to the simultaneous equations

$$y = 2x + 7$$

$$2y - 4x = 16$$

Triple Simultaneous Equations

Solve:

$$5x + 3y + z = 24$$

$$4y + 2z = 16$$

$$3z = 18$$

Mean Problem

x, y and z satisfy

$$x + y + 3z = 121$$

$$x + 3y + z = 678$$

$$3x + y + z = 356$$

Find the mean of x, y, z, without using a calculator

Hint:

- Write an expression for the mean of x, y, z
- Do you need to find x, y, z seperately to find the mean?

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Linear Sketching

Did you know?

Where is the steepest street in the world?

Gradients can be represented in different ways, but what do the measurements mean?

A gradient of $1:5$ means for every 5m you travel horizontally you travel 1m vertically.

A gradient of 16% means for every 100m across you go 16m up.

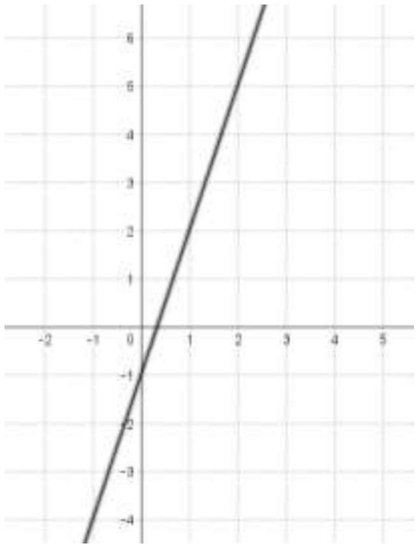
Can you find out:

- Where the street is?
- What the gradient of the street is?
- Why there is controversy over the winners?

Clue: They hold a Jaffa rolling contest down the street every year!

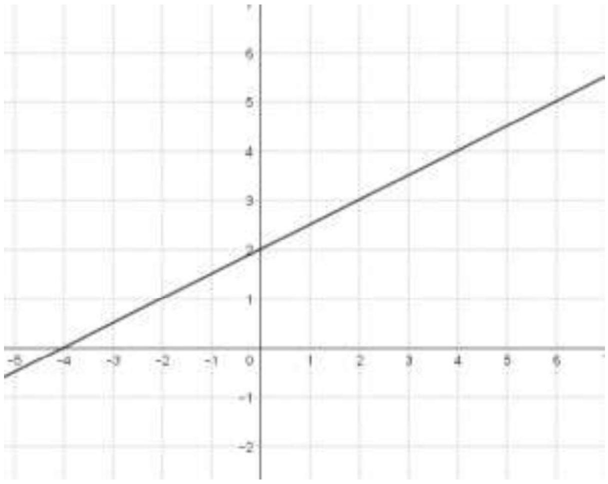
Linear Graphs 1

1. What are the gradient and intercept of the line $y = 3x - 5$?
2. Find the gradient of the line connecting (3,10) and (1,6)
3. Find the midpoint between the points (3,-8) and (-1,4)
4. Find the distance between points (1,10) and (4,18)
5. What is the equation of the line with gradient 3 that passes through (5,8)?
6. Does the line $y = 2x - 3$ pass through (1,-1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = 5x - 2$ that passes through (2,19)
8. What is the equation of this graph?



Linear Graphs 2

1. What are the gradient and y intercept of the line $y = 2x - 7$?
2. Find the gradient of the line connecting (1,4) and (-1,0)
3. Find the midpoint between the points (-2,10) and (6,4)
4. Find the distance between the points (4,11) and (-1,15)
5. What is the equation of the line with gradient 2 that passes through (1,4)?
6. Does the line $y = -2x + 5$ pass through (3,1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = -\frac{3}{2}x - 1$ that passes through (6,4)
8. What's the equation of this graph?



Do they cross?

Line A passes through the points $(-3,1)$ and $(3,5)$
Line B passes through the points $(0,-4)$ and $(6,4)$

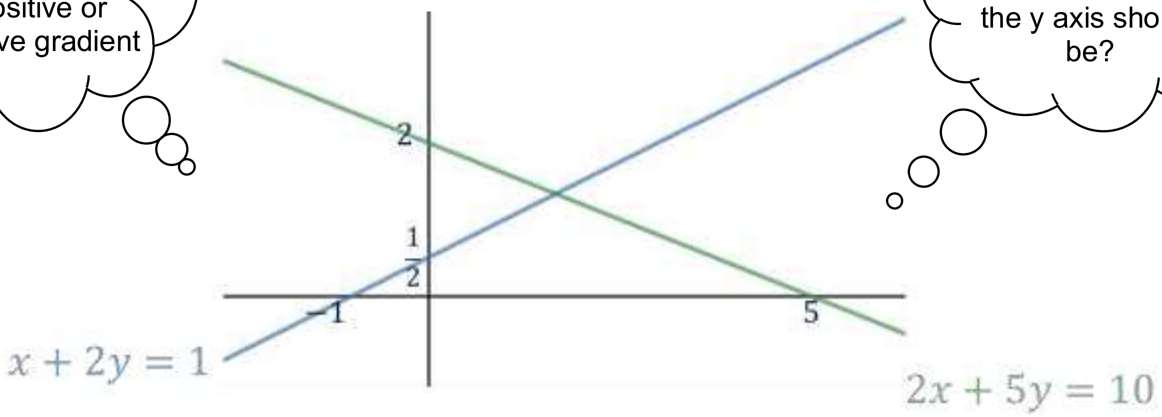
- By sketching can you tell if the lines will meet?
- If they do meet what are the points of intersection?
- Challenge! Can you find where the lines will meet using algebra

Picture this

Think about whether the equation suggests a positive or negative gradient

Is this an accurate sketch of these two lines?

What does the equation suggest the intercept with the y axis should be?



The plot thickens….

Complete the information in the table for each equation below:

- Find the co-ordinates of the x and y intercepts
- Decide if the gradient of the graph would be positive or negative

Name	Equation	x intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$			
B	$y = 3$			
C	$3x + 4y = 2$			
D	$2x - y + 6 = 0$			
E	$2y + x = 4$			
F	$2x + y - 3 = 0$			

Using the information from the table, sketch all the graphs on one set of axes to find:

- A pair of lines that are parallel
- A pair of lines that are perpendicular
- A pair of lines that intersect at $(-2,\ 2)$

Two geometry problems

DEF is an isosceles right angled triangle
The line passing through D and F has the equation $x + 3y = 15$
D is the co-ordinate $(6,3)$
E is the co-ordinate $(5,0)$
The angle EDF is the right angle

Can you find:

- The equation of line DE?
- The possible coordinates of F?
- The equation of line EF?

ABCD is a parallelogram. The line passing through C and D has the equation $y = 7$
The line CD is 5 units long
D has coordinate $(2,7)$
C has both positive x and y co-ordinates
The line through AC has equation $3x + 2y = 35$
A has coordinate $(9,4)$
Can you find:

- The coordinate of C?
- The equation of line AB?
- The equation of line BD?
- The area of the parallelogram?

Sketching Linear Inequalities

Sketch and shade the following inequalities.

1. $y \leq 6$
2. $x < 6$
3. $x + 2y \geq 8$
4. $3x + 2y \geq 12$

- Shade out the side of the line that doesn't satisfy the inequality.
- Label the correct region **R**

Geometry from equations

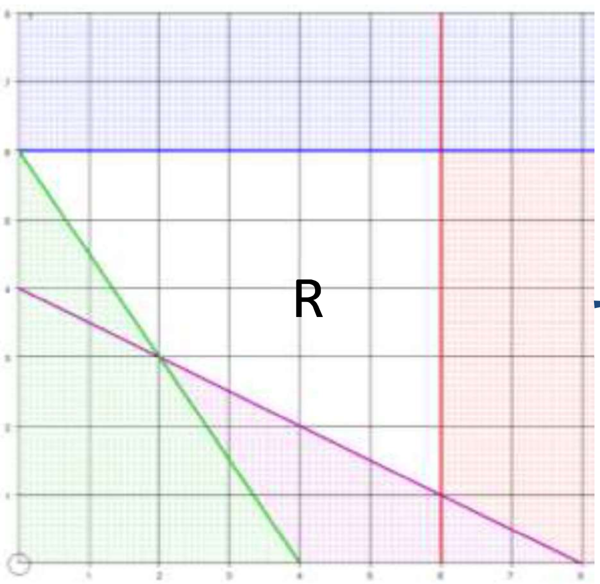
The following equations enclose a square:

$$y - 2 = x$$
$$y + x = 6$$
$$y = x - 1$$
$$y + x - 3 = 0$$

- Which are the two pairs of parallel sides?
- What are the coordinates of all 4 vertices
- How can you convince yourself this is a square?

Linear Programming

Here is a graph that shows the feasible region R satisfied by the all inequalities from the previous question.



In Linear Programming linear inequalities are used to find solutions to real life problems.

The 'optimal' or best solution for is found for a particular objective.

R, the unshaded region is called the FEASIBLE REGION. Points in this region satisfy all of the inequalities.

The feasible region has four vertices What are the coordinates?

- Use the diagram to have a go at this question

Maximise the value of $x + y$ within the region satisfied by the inequalities:

$$x + 2y \geq 8, \quad 3x + 2y \geq 12, \quad y \leq 6, \quad x \leq 6$$

Catching Stars

Go to Student.Desmos.com (use classroom code: **3VJUM2**) to try a Linear Marbleslides Challenge.

You will be investigating the features of linear graphs whilst trying to catch as many stars as possible.

You can join the activity without signing in or entering your real name.

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More Brackets

Did you know?

Have a think about the expression (x + 2)(x + 1)

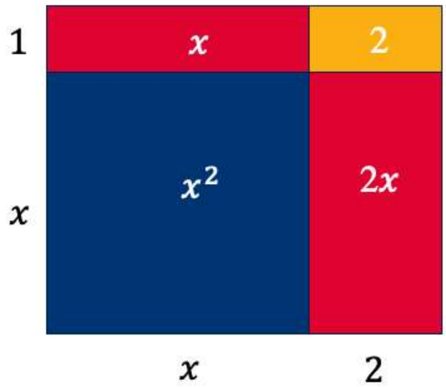
Formal Method	Geometrical Representation	Grid Method

Formal Method

$$(x+2)\,(x+1)$$
$$= x\,(x+1)+2(x+1)$$
$$= x^2+x+2x+2$$

$$= x^2+3x+2$$

Geometrical Representation



$$= x^2+3x+2$$

Grid Method

	x	$+2$
$+1$	x	2
x	x^2	$2x$

$$= x^2+3x+2$$

- This expression expands to give 4 terms…which simplify to 3 terms.
- How many terms are in the un-simplified expansion of $(x+3)(x+4)(x+5)$?
- Be prepared to explain your thinking…

The story so far….

1. Expand and simplify
$$(2x+3)(3x-5)$$

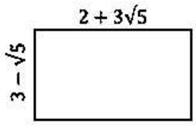
2. Write $(x+3)^2-4$ in the form ax^2+bx+c

3. Expand and simplify
$$(2a+2)(3x-4a+3)$$

4. Expand and simplify
$$3x(x-3)(x+5)$$

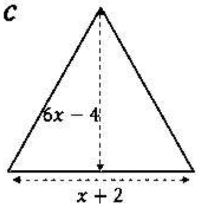
5. Evaluate (no calc allowed)
$$\left(2+\frac{1}{3}\right)\left(2-\frac{1}{3}\right)$$

6. Find the area of this rectangle



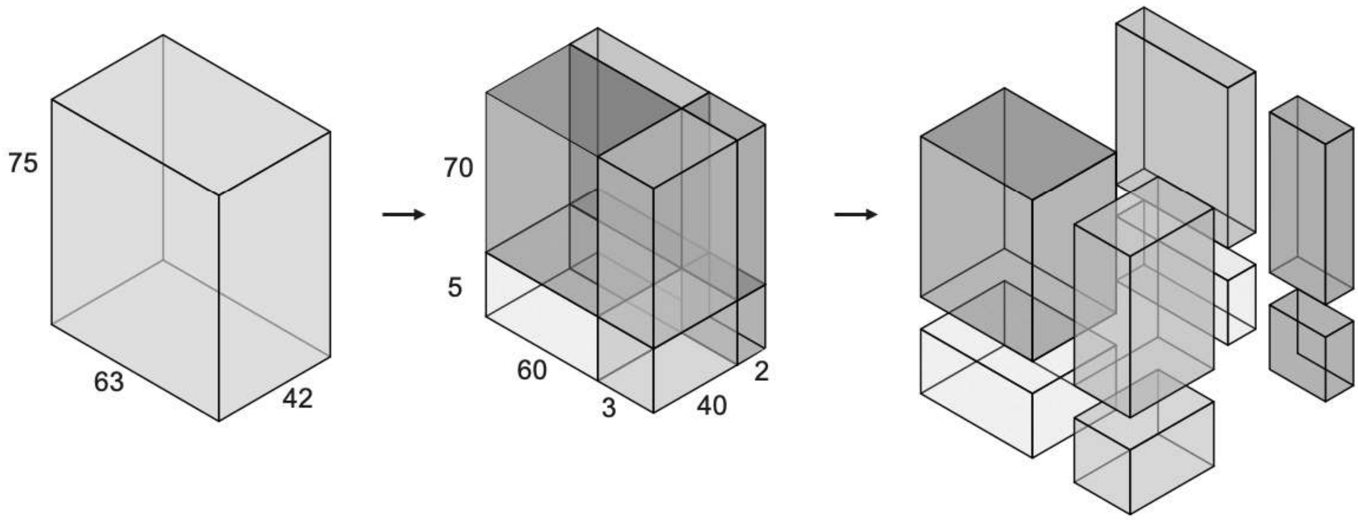
7. Expand and simplify
$$(5-4x)(3x+6)+(5x-2)(3+4x)$$

8. Find the area of the triangle and write it in the form ax^2+bx+c



Have a look…

These diagrams represent a method for calculating 63 x 42 x 75



- Can you see what is going on?
- What are the values of the coloured sections?
- This diagram should help you answer the question: “How many terms there are in an un-simplified expression $(x+3)(x+4)(x+5)$?”

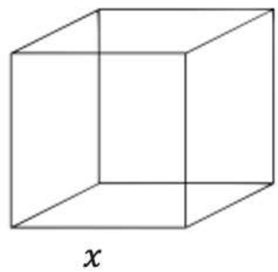
Think again

How might you go about expanding the following?

$$(x+2)(x+1)(x+2)$$

- Is there a way you could use some of the previous workings?
- How would the geometric diagram have to change?
- Could you use a grid method to speed things up?

Getting Bigger



Here is a cube with side lengths of x cm

The cube is going to have its lengths increased in one of three ways

Method A	Method B	Method C
Each side is increased by 2 units	One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit	One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

- Can you prove which of the solids will have the largest volume?

Expanding Cubics and Beyond

Previously we saw how we could use a grid for expanding brackets such as (1 + x)^2

	1	$+\boldsymbol{x}$
1	1	$+x$
$+\boldsymbol{x}$	$+x$	$+x^2$

So $(1+x)^2 = 1 + 2x + 1x^2$

	1	$+\boldsymbol{2x}$	$+\boldsymbol{x^2}$
1	1	$+2x$	$+x^2$
$+\boldsymbol{x}$	$+x$	$+2x$	$+x^3$

So $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$

Can you use a similar approach to expand:

- $(1+x)^4$
- $(1+x)^5$
- $(1+x)^6$

Pascal's Triangle 1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

- Look carefully at this triangle of numbers – have you seen it before?
- Can you work out any patterns that are present?
- Take a careful look at the coefficients that you have found in the previous task.
- Can you see a connection?

Pascal's Triangle 2

Here's the same triangle, referenced with the coefficients of a (1 + x)^n expansion

Row 0: $(1+x)^0 = 1$ → 1

Row 1: $(1+x)^1 = 1 + x$ → 1 1

1 2 1

Row 3: $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$ → 1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Row 6: $(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + 1x^6$

Use the triangle to help you complete these expansions:

$(1+x)^7$ $\qquad\qquad$ $(1+x)^8$

Summary and review

1. Expand and simplify
   $\left(\frac{1}{3}x + \frac{1}{9}\right)\left(3x - \frac{2}{3}\right)$

2. Expand and simplify
   $(x+1)(x+2)(x+3)$

3. Expand and simplify
   $(x-3)(x+2)^2$

4. Expand and simplify
   $(2 - \sqrt{3})(1+\sqrt{3})(1-\sqrt{3})$

5. Find the volume of a cube with side length $x - 4$

6. Expand and simplify
   $(x^2-2)(x^2+2)(x+1)$

7. Write $\left(\sqrt{y} + \sqrt{8y}\right)^2$ in the form $a + b\sqrt{2}$.

   Given that $\left(\sqrt{y} + \sqrt{8y}\right)^2 = 54 + b\sqrt{2}$.
   Find values for y and b.

8. Simplify $\frac{(x-1)(x+2)}{(x+3)} - \frac{4}{2x+1}$

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Further Factorising

Did you know?

Substitute $x = 9$ into the following two expressions

$$\boldsymbol{x^2 + 3x + 2}$$

What do you notice?

$$(9)^2 + 3(9) + 2 = 81 + 27 + 2 = 110$$

$$\boldsymbol{(x + 2)(x + 1)}$$

$$(9 + 2)(9 + 1) = 11 \times 10 = 110$$

Both give the same answer as the expressions are equivalent

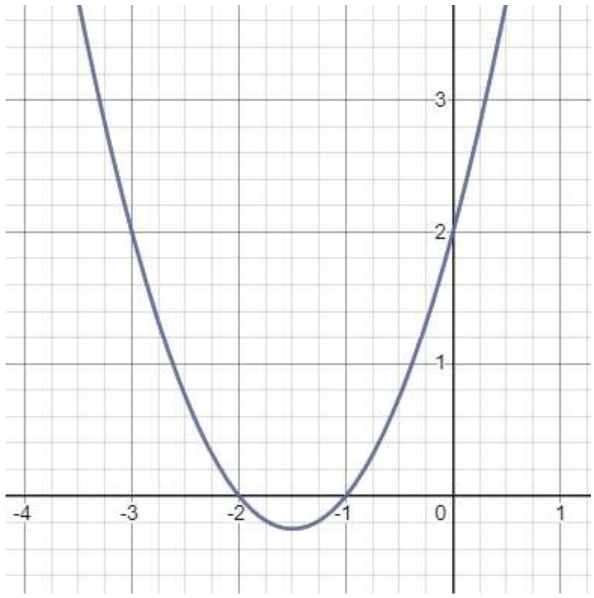
One of the expressions was a lot easier to evaluate! Why?

Which is best?

$x^2 + 3x + 2$ *or* $(x + 2)(x + 1)$

expanded form — factorised form

$y = x^2 + 3x + 2$ $y = (x + 2)(x + 1)$



Factorising is a key skill for both sketching graphs and solving equations, both of which will be covered later.

Sometimes it is more helpful to factorise an expression, other times better to be expand it, depending on the context.

Further Factorising 1

Factorise the following fully:

1. $x^2 + 5x - 6$
2. $x^2 + 13x - 30$
3. $y^2 - 13y + 30$
4. $t^2 + 2t - 15$
5. $k^2 - 2k - 24$
6. $p^2 - 10p + 21$
7. $x^2 - 16x$
8. $3x(2x - 1) + 4(1 - 2x)$

Further Factorising 2

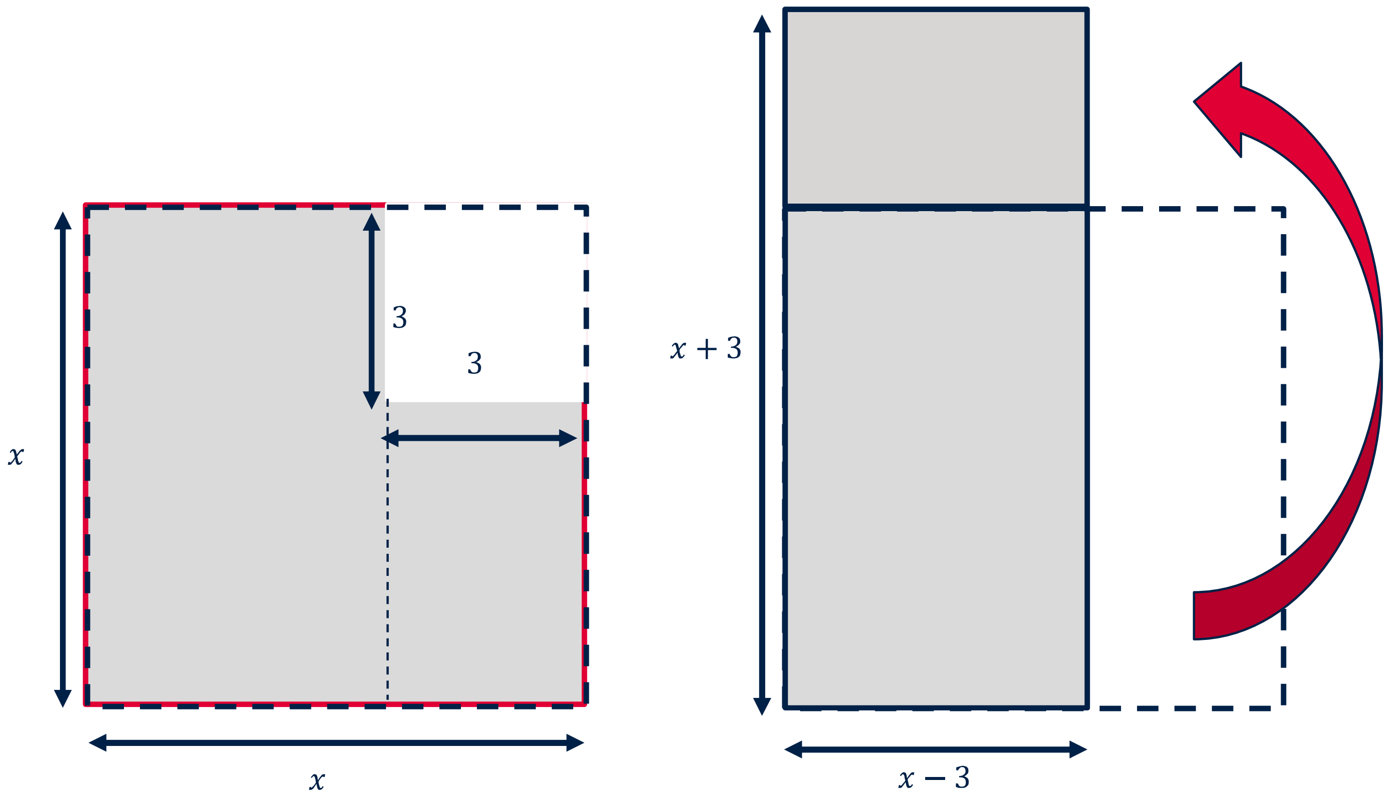
Factorise the following fully:

1. $x^2 + 6x - 7$
2. $y^2 + y - 12$
3. $y^2 - 11y + 28$
4. $t^2 + 7t - 18$
5. $k^2 + 9k + 20$
6. $x^2 + x - 56$
7. $p^2 - 25p$
8. $x^2(3x - 4) + (4 - 3x)$

Difference of Two Squares

A special case for factorising is the difference of two squares.

Expressions such as $x^2 - 3^2$, where the coefficient of x is zero.



$$x^2 - 3^2 = (x-3)(x+3)$$

Try factorising these expressions using the difference of two squares

1. $x^2 - 6^2$
2. $y^2 - 144$
3. $x^2 - y^2$
4. $4t^2 - 81$
5. $x^2 - 5$

$ax^2 + bx + c$

So far we have been factorising quadratic expressions where $a = 1$. For example, $x^2 - 2x - 15$

Time to try some trickier quadratics!

Have a go at this one…

Factorise $\mathbf{6x^2 + 19x + 10}$

If you got $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$ Well done!

Feeling confident? You can try the Trickier Quadratics questions below

There are many methods for factorising quadratics where $a > 1$

You might want to refresh your memory on the method that you learnt at school if you are going to tackle the following questions.

Trickier Quadratics

1. $\mathbf{3x^2 - 10x - 8}$
2. $\mathbf{2x^2 - 7x + 6}$
3. $\mathbf{4y^2 + 20y + 9}$
4. $\mathbf{6x^2 - 13x - 8}$
5. $\mathbf{20x^2 + x - 12}$

Further Factorising Problems

These expressions are slightly different to the previous ones but can still be factorised.

1. $2t^2 - 32$
2. $x^3 - 7x^2 + 12x$
3. $x^4 - x^2 - 2$
4. $y^4 - 625$

Without a calculator

What is the value of each of the following?
calculators not allowed

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Still without a calculator

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Top and Bottom

Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Some possible hints!

Without a calculator Hint	Still without a calculator Hint	Top and Bottom Hint
■ Can you factorise $9^2 - 1^2$? ■ How does this help?	■ Replace 123 by n and 122 by $n-1$ ■ Now go on to factorise	■ Factorise the numerator then the denominator ■ What do you notice?

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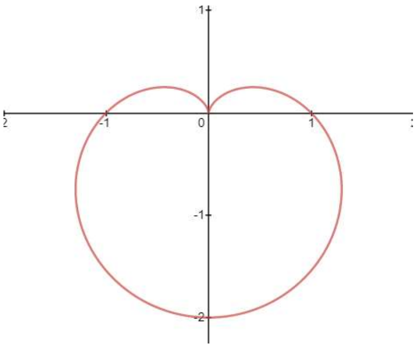
Rearranging Factorising

Did you know?

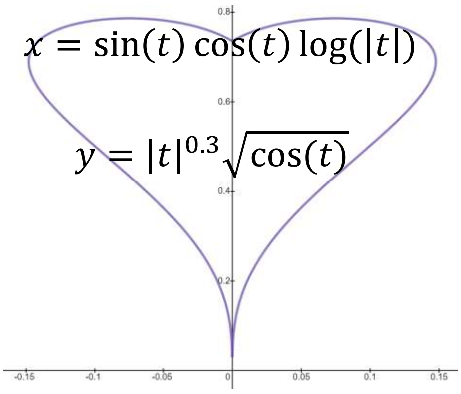
- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these

$r = 1 - sin\theta$

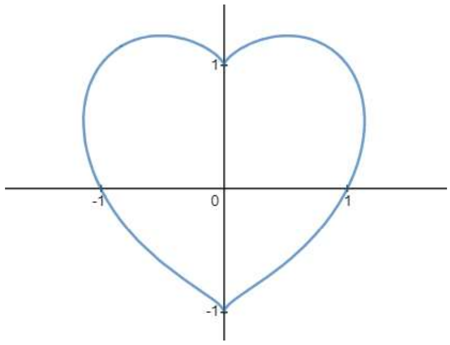


$x = \sin(t)\cos(t)\log(|t|)$

$y = |t|^{0.3}\sqrt{\cos(t)}$



$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$



Futher Factorising 1

1. The equation of a line is given as
   i. $3y + 4x - 2 = 0$.
   b. What is the gradient of the line?

2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x

3. Make x the subject of:
   a. $ax - y = z + bx$

4. The equation of a line is given as
   i. $5(\mathrm{b} - \mathrm{p}) = 2(\mathrm{b} + 3)$

5. John says the first step to rearranging
   a. $\frac{x-a}{f} = 3g$ is to add a *to* $3g$. Is he right? Explain your answer.

6. Make a the subject of
   a. $5(a - t) = 3(a + x)$

7. Make x the subject of
   a. $ay + x = 4x + xb$

8. Make x the subject of
   a. $2\pi\sqrt{x + t} = 4$

Further Factorising 2

1. Make y the subject of
   $xy + 6 = 7 - ky$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of
   $\frac{4}{y} + 1 = 2x$

5. Displacement can be expressed as
   i. $s = ut + \frac{1}{2}at^2$

   Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula
   i. $A = \frac{1}{2}\left(\frac{a+b}{h}\right)$

   Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$

Equivalent quadratics

Sort the expressions below in to 4 sets of 4 equivalent expressions

$x^2 - 25$	$2x^2 - 2$
$(x+5)(x+6) - x - 55$	$(x+5)(x-5)$
$2(x^2 - 1)$	$(x+5)^2 - 10x - 50$
$2(x+3)(x-1)$	$2(x+1)(x-1)$
$(x+5)^2 - 50$	$2(x+2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x+5)(x-5) + 10x$
$2(x+1)^2 - 8$	$(x-5)(x+6) - x + 5$
$x^2 + 10x - 25$	$2(x+1)^2 - 4(x+1)$

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

THEN

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

Hint

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Difference of numeric squares

Problem 1

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

"Well, that's just $15^2 - 9$ which is 216"

Mr Lo was amazed.

- **How did she know so quickly what the answer was?**

Problem 2

Use the fact that 3 x 4 = 12

Can you quickly work out a value for $(3.5)^2$?

- **Can you see a connection between the previous question and this one?**

The Quadratic Formula

We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?
- Can you prove why the quadratic formula works?

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	$ax^2 + bx = -c$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Match the steps below with the algebra above for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

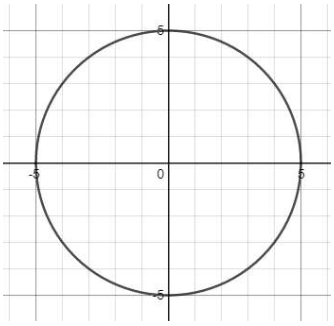
Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

Equations of Circles

$$x^2 + y^2 = 25$$

Represents a circle with centre (0,0) and radius 5



Generally, the equation of a circle with centre (0,0) and radius r can be written as

$$\boldsymbol{x^2 + y^2 = r^2}$$

- **What happens if the centre is not (0,0)?**

Let's have a look at this equation: $x^2 + 4x + y^2 - 6y = 12$

We can rearrange this by completing the square separately for the x terms and y terms

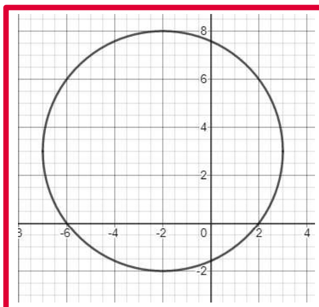
$$x^2 + 4x = (x+2)^2 - 4 \text{ and } y^2 - 6y = (y-3)^2 - 9$$

So $x^2 + 4x + y^2 - 6y = 12$

Can be written as $(x+2)^2 - 4 + (y-3)^2 - 9 = 12$

$$(x+2)^2 + (y-3)^2 - 13 = 12$$

$$(x+2)^2 + (y-3)^2 = 25$$



$$(x+2)^2 + (y-3)^2 = 25$$

Represents a circle with Centre (-2,3) and radius 5

- Can you find the centre and radii of these circles by rearranging into the form

$$(x+a)^2 + (y-b)^2 = r^2$$

$x^2 - 8x + y^2 - 2y = 19$	$x^2 + 6x + y^2 - 10y = 15$

Solving Quadratics

Did you know?

I have picked two numbers that multiply to make zero.

What can you say about my numbers?

At least one of them must be zero

This is useful when using factorising to solve equations.

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both!)

Historically zero wasn't accepted as a number until fairly recently!

Solving with Quadratics 1

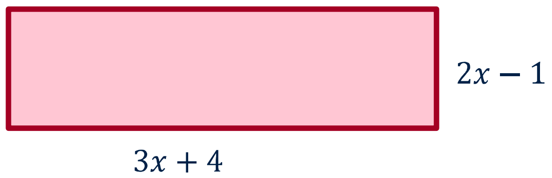
Solve the following

1. $x^2 = 16$
2. $x^2 - 16x = 0$
3. $(x+1)(2x-3) = 0$
4. $x^2 - 3x + 2 = 0$
5. $(2x-5)(4x+3) = 0$
6. $3x^2 + 14x - 5 = 0$
7. $(x+3)^2 = 25$
8. $\frac{3}{x} + \frac{4}{x-1} = 10$

Solving with Quadratics 2

Solve the following

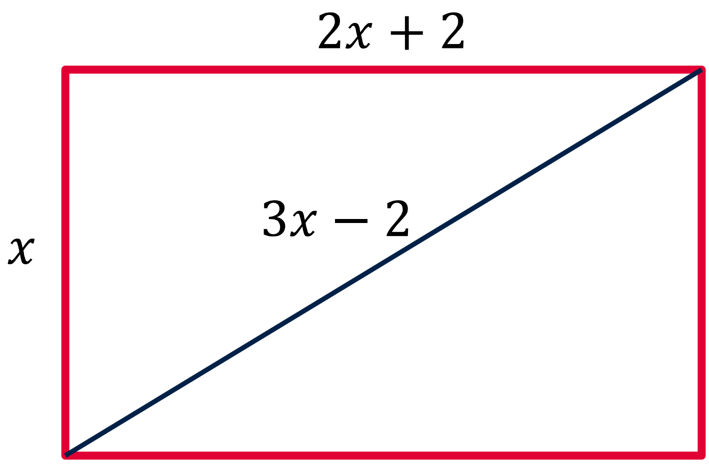
1. $x^2 - 4x - 12 = 0$
2. $x^2 - x = 6$
3. $2x^2 - 11x + 12 = 0$
4. $6x^2 + x - 12 = 0$
5. $3 + 2x - x^2 = 0$
6. $x^2 - 4x - 1 = 0$
7. $\frac{8}{x+2} - \frac{14}{x-3} = 9$
8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$
b) Find any possible values for x

Quadthagoras

Find the length, width and diagonal of this rectangle



Up in the air!

An object is launched from a cliff that is $58.8m$ high.

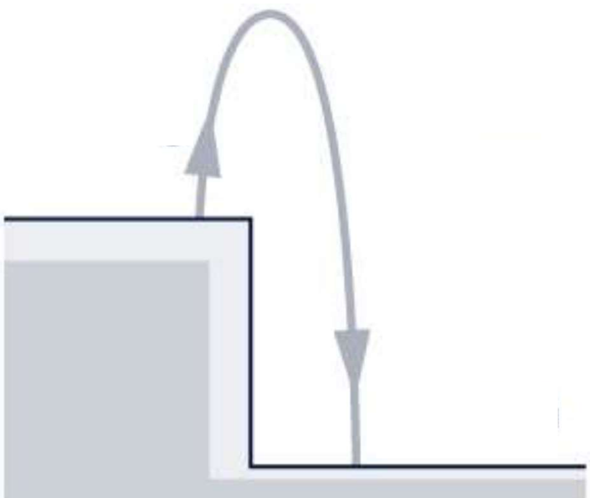
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height *h* above the ground at time *t* seconds after launch is

$$h = -4.9t2 + 19.6t + 58.8$$

where *h* is in metres.

- When does the object strike the ground?



Which Way?

In the skills check you saw how we can solve quadratic equations by factorising or completing the square.

We can also use the quadratic formula. For a quadratic $ax^2 + bx + c = 0$ the solutions are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try solving $x^2 + 4x - 21 = 0$ using each of the three methods.

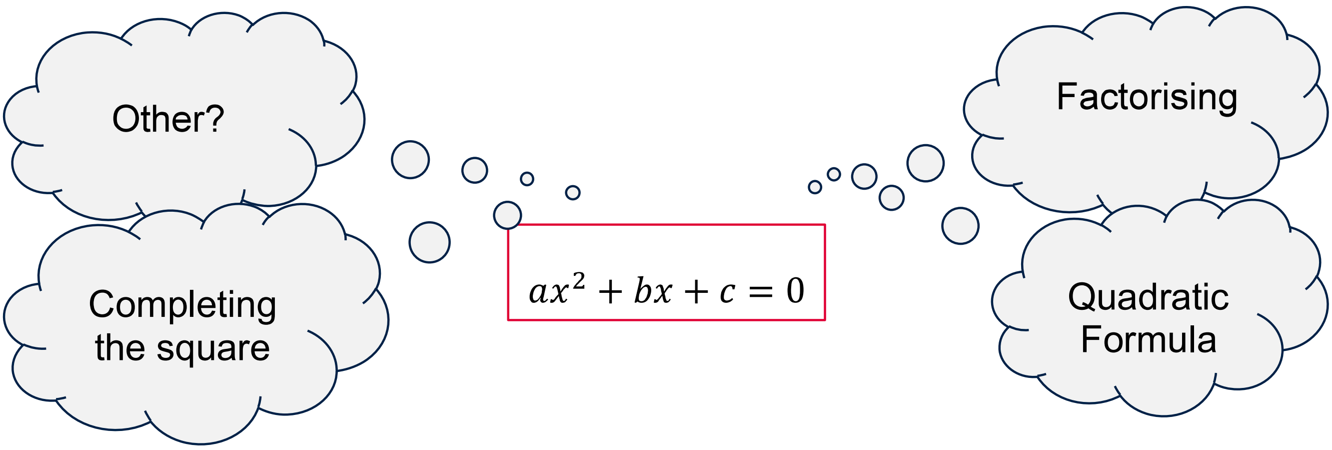
Try solving $3x^2 + 4x - 2 = 0$ using each of the three methods.

Which Way Now?

There is not always one best way to solve a quadratic.

Some methods are better than others for different equations

How can you spot which is the right method for each equation?

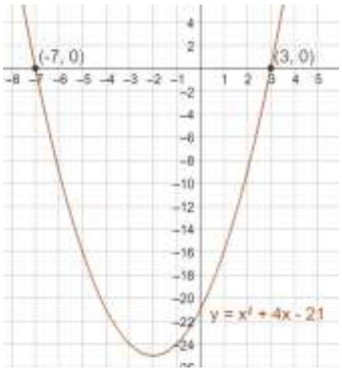


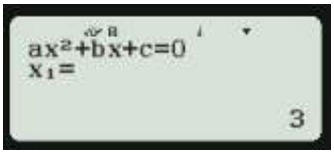
https://undergroundmathematics.org/quadratics/quad-solving-sorter is a really good activity for improving your skills in sorting quadratic equations. You or your teacher may be able to print the cards out to help.

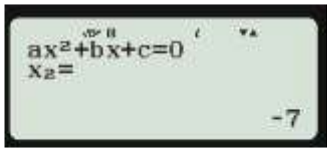
Another Way?

And of course, there are the methods of solving using graphs and/or your calculator

$$x^2 + 4x - 21 = 0$$







Using Graphs

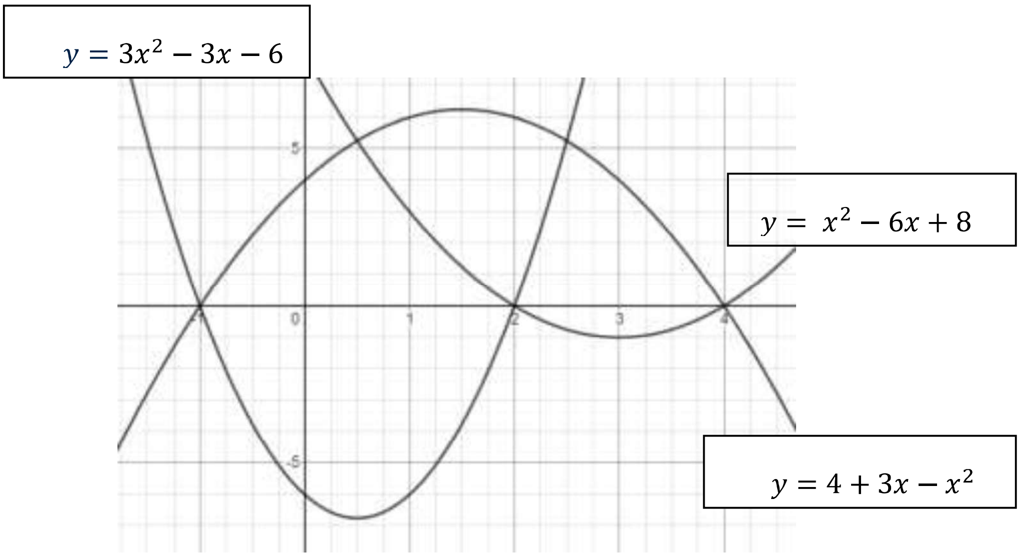
Use the graphs to solve

$$4 + 3x - x^2 = 0$$

$$x^2 - 6x + 8 = 0$$

$$3x^2 - 3x - 6 = 0$$

$$4 + 3x - x^2 = 4$$



Simultaneously

Solve these pairs of equations

1. $y = x^2 + 6x - 9$
$y = 3x + 1$

2. $y = x^2 + 2x + 2$
$y - 4x = 1$

3. A rectangle has length $(a + b)$ and width $3a$.
The area is $60cm^2$ and perimeter is $32\ cm$.
Calculate, algebraically, the values of a and b.

4. In how many places does the line $y = 2x + 2$ intersect the circle $(x + 2)^2 + y^2 = 25$?
What are the co-ordinates of these intersections?

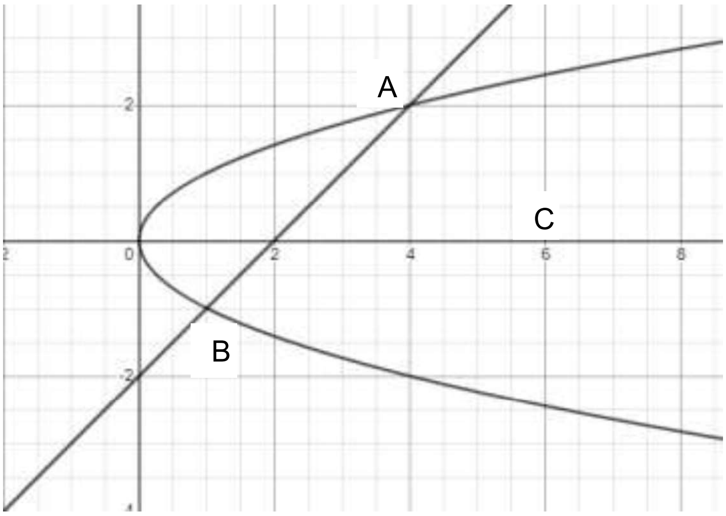
Lines and Curves

The diagram shows the graphs of

$y^2 = x$ and $y = x - 2$

The graphs cross at the points A and B.

The point C has co-ordinates (6,0)



- Without the use of a calculator, find the exact area of triangle ABC