Transition Materials

GCSE Maths to A-Level Maths

Pack B

Contents:

Produced by the Advanced Mathematics Support Program for the Arthur Terry School

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Advanced Mathematics Support Programme

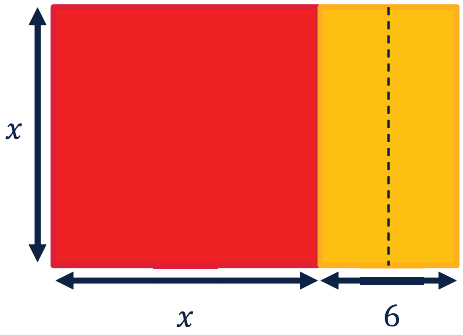
Completing the Square

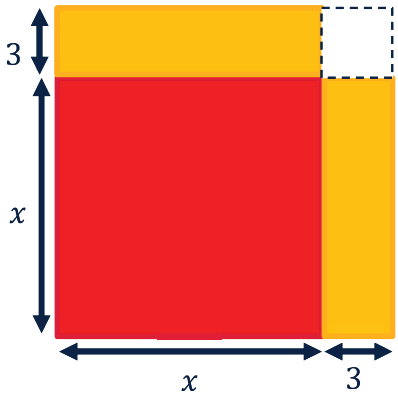
Did you know?

These are different forms of the same algebraic expression

$$x^2 + 6x \quad = \quad x(x+6) \quad = \quad (x+3)^2 - 9$$

expanded form | factorised form | completed square form





Do the diagrams help you see why this is called **Completing the square**?

Completing the square 1

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 4x$
2. $x^2 + 4x + 5$
3. $y^2 - 8y$
4. $y^2 - 8y + 7$
5. $x^2 - 12x + 41$
6. $k^2 + 10k - 2$
7. $y^2 + 3y + 1$
8. $p^2 - 2p + 1$

Completing the square 2

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 10x$
2. $x^2 + 10x + 30$
3. $y^2 - 2y$
4. $y^2 - 2y + 3$
5. $x^2 - 8x + 25$
6. $k^2 + 14k - 1$
7. $y^2 + 5y + 6$
8. $t^2 + 6t + 9$

Extra Puzzle 2

Given that

$$55^2 - 45^2 = (55 + 45)(55 - 45) = 1000$$

and

$$60^2 - 40^2 = (60 + 40)(60 - 40) = 2000$$

- Find numbers a and b such that $a^2 - b^2 = 3000$
- Find numbers c and d such that $c^2 - d^2 = 4000$
- Find numbers e and f such that $e^2 - f^2 = 100\,000$

Advanced Mathematics Support Programme

Sketching Quadratic Graphs

Did you know?

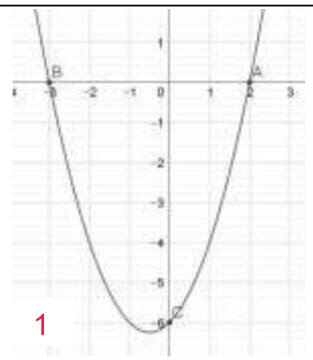
Quadratic curves are also known as parabolas.

Parabolas are used in many examples of architecture.

- Do you recognise these landmarks?
- Do you think they are parabolas?
- Can you find any more examples of architecture that use parabolas near where you live?

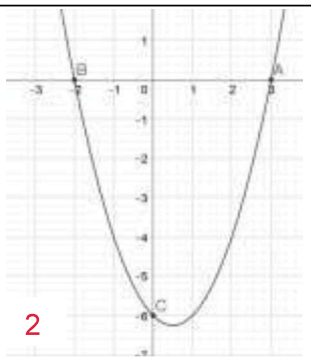
Quadratic Graphs 1

Find the coordinates of A, B and C on each graph



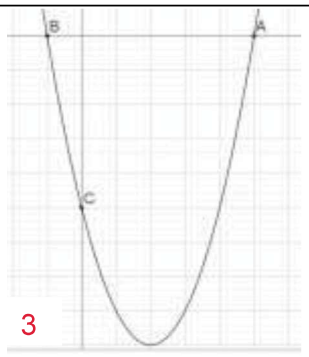
1

$y = (x + 3)(x - 2)$



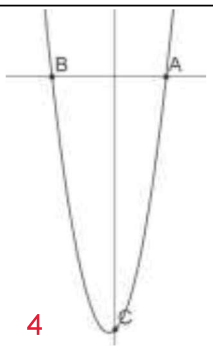
2

$y = (x - 3)(x + 2)$



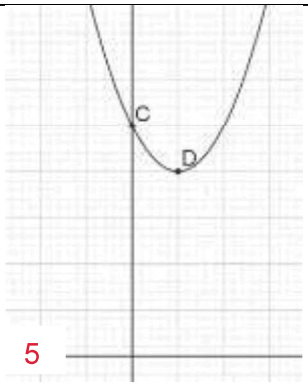
3

$y = (x - 5)(x + 1)$



4

$y = (x + 5)(x - 4)$

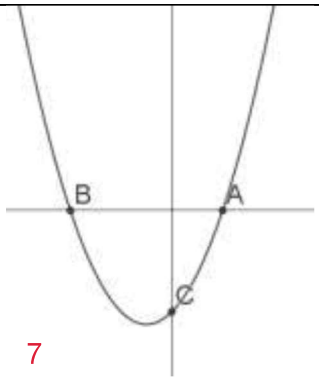


5

$y = (x - 1)^2 + 4$

6

$y = (x + 3)^2 + 7$



7

$y = x^2 + 3x - 4$

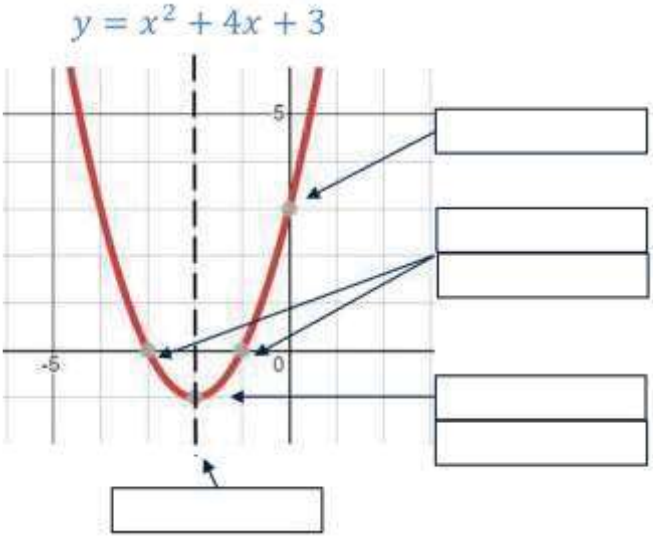
8

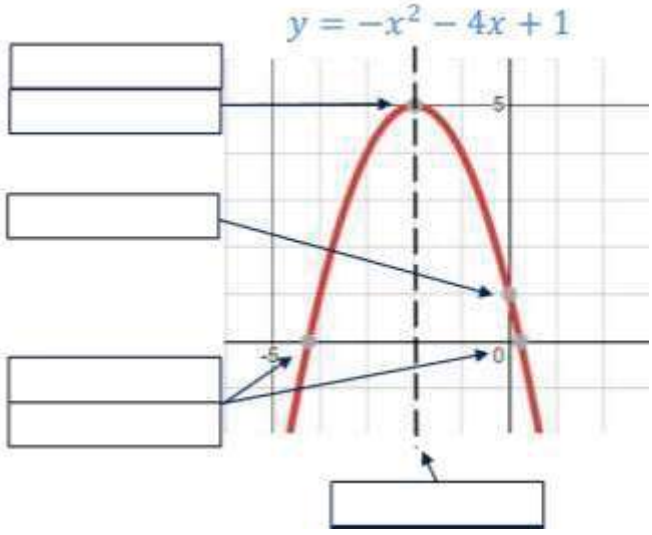
$y = 2x^2 + 4x - 6$

What is a sketch?

In mathematics a sketch does not need to be a completely accurate drawing, but it should "illustrate all the significant features of the graph/shape"

These diagrams show the key features of a quadratic graph





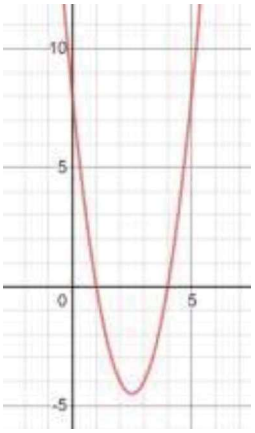
Put the words below into the boxes above so that the quadratic graphs are labelled correctly. Some words may be used more than once.

x intercepts	minimum	roots	turning point
maximum	axis of symmetry	y intercept	

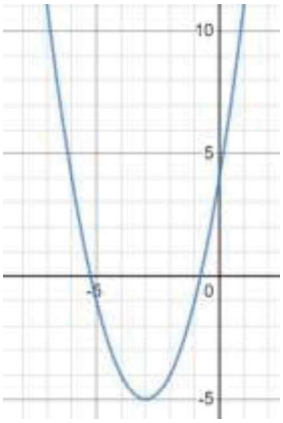
Identification Parade

Which of the graphs below is $y = x^2 - 5x + 4$?

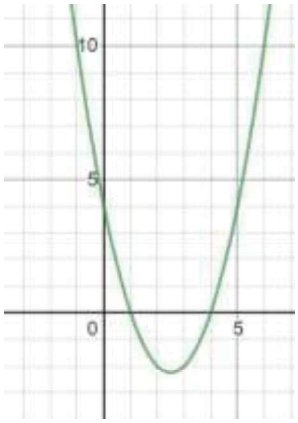
A



B



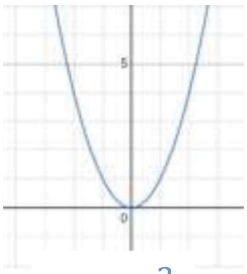
C



Move it!

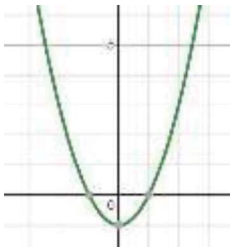
- Can you describe how to move Graph A onto Graph B?

GRAPH A



$y = x^2$

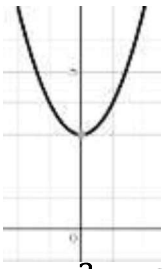
GRAPH B



$y = x^2 - 1$

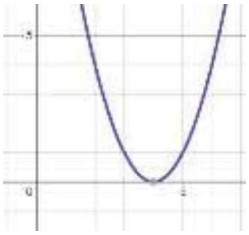
- Which transformations would take GRAPH A onto each of the graphs below?

GRAPH C



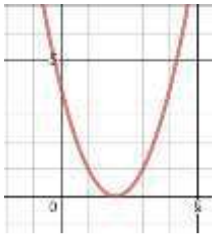
$y = x^2 + 3$

GRAPH D



$y = (x - 4)^2$

GRAPH E

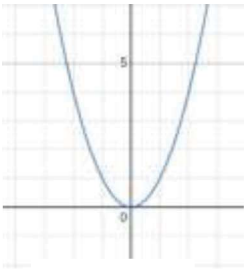


$y = (x - 2)^2$

Move it again!

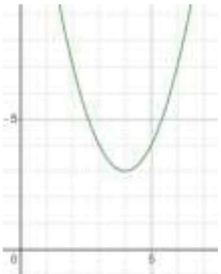
- Can you describe how to move Graph A onto Graph F?

GRAPH A



$y = x^2$

GRAPH F



$y = (x-4)^2 + 3$

- Can you see how that links to the equation of the graph?

Complete the square to get sorted!

Below are ten quadratic equations.

$y = x^2 - 4x + 7$ $\quad$ $y = x^2 + 2x + 5$ $\quad$ $y = x^2 - 6x + 16$ $\quad$ $y = x^2 - 6x + 25$

$y = x^2 - 2x + 5$ $\quad$ $y = x^2 + 8x + 20$ $\quad$ $y = x^2 - 6x + 11$ $\quad$ $y = x^2 - 8x + 21$

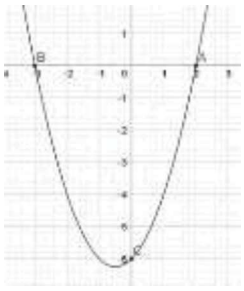
$y = x^2 + 6x + 10$ $\quad$ $y = x^2 - 10x + 29$

Your task is to place **nine** of them into a 3 by 3 grid according to the rules on these cards

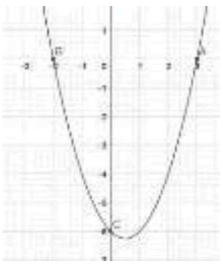
All of the equations in the top row have a turning point on the line $y = 4$	One of the equations in the left hand column has its turning point at (-4, 4)	The equation with a turning point at (-1, 4) is not on the top row	The equations in the top left and centre right square both have the same y coordinate for their turning point	All of the coordinates of the turning points are at integer values of x and y. None of the turning points are on either axis.
All three of the equations with a turning point on the line $x = 3$ are on the bottom row	All of the equations in the centre column have turning points on the line $y = 5 - x$	All of the turning points for the equations in the centre column are in the first quadrant	$y = x^2 + 6x + 10$ is in the square in the left hand column directly above $y = x^2 - 6x + 16$	The turning point of the equation $y = (x+a)^2 + b$ is at $(-a, b)$

Quadratic Graphs 2

1. What are the x intercepts of $y=(2x+3)(x+4)$?
2. What are the x and y intercepts of this graph.



3. Write the equation of the graph in the form ax^2+bx+c



4. What are the x intercepts of the graph of $y=6x^2+x-2$?
5. What does the c part of the equation in $y=ax^2+bx+c$ represent on a graph?
6. Sketch the graph of $y=3x^2-2x-8$ Label the x and y intercepts
7. What are the coordinates of the points marked on the diagram of the equation $y=x^2+6x+16$?

8. Which of these statements about the graph $y=x^2-4x+8$ are true

Has a minimum point at (2, 4)	Will not cross the x axis twice	Can be factorised

How High?

The height of a ball thrown up from the ground into the air at time, t, is given by:

$$h=20t-10t^2$$

- Find when the ball hits the ground
- How long is the ball more than 5m above the ground?
- Find the maximum height reached by the ball

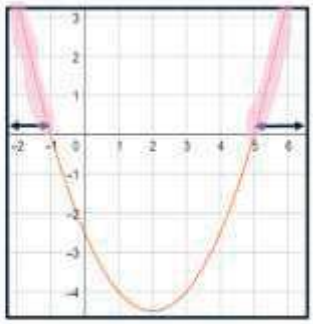
Inequalities reminder

Solve the inequality $x^2 - 4x - 5 \geq 0$

Rearrange into factorised form...

$(x - 5)(x + 1) \geq 0$

...now you can sketch the graph



When $x < -1$ and $x > 5$
the curve is above the x axis.

This is where $x^2 - 4x + 5 \geq 0$

These are two regions, so are represented by two inequalities

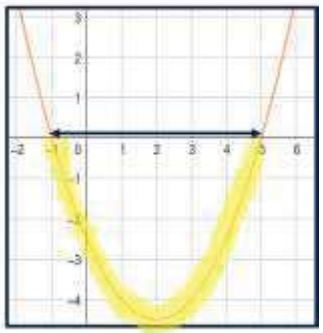
$x < -1$ and $x > 5$

Solve the inequality $x^2 - 4x - 5 < 0$

Rearrange into factorised form...

$(x - 5)(x + 1) < 0$

...now you can sketch the graph



When $x > -1$ and $x < 5$
the curve is below the x axis.

This is where $x^2 - 4x + 5 < 0$

This is one region, so can be represented by one inequality

$-1 < x < 5$

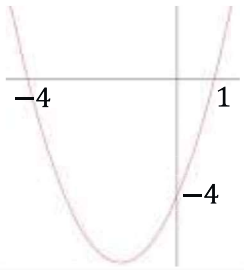
Quadratic Inequalities

- Use a sketch to help you solve the following inequalities

1. $(x - 2)(x + 3) < 0$
2. $(4 + x)(2 - x) < 0$
3. $x^2 + 7x + 12 \geq 0$
4. $36 \geq (x + 2)^2$

Fill the table

- Complete the rows in the table with the information that you have been given

Sketch	Equation	x intercept	y intercept	Minimum point
				
		$(5,0)$ $(-2,0)$	$(0,-10)$	
				(-5, 6)
	$y = x^2 + 6x + 8$		$(0,8)$	

Catching Stars

Go to Student.Desmos.com (use classroom code: **E96QV4**) to try a Quadratic Marbleslides Challenge.

You will be investigating the features of quadratic graphs whilst trying to catch as many stars as possible.

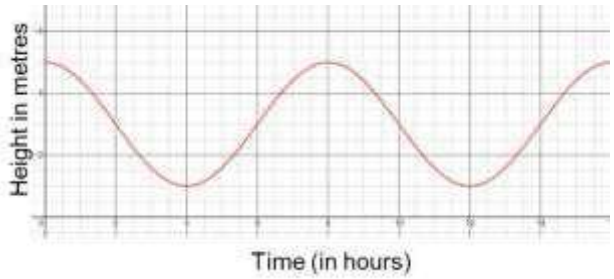
You can join the activity without signing in or entering your real name.

Sketching Other Graphs

Did you know?

Trigonometric functions can be used to model many things that repeat over a time period.

Examples include: Tides, springs, harmonic strings and hours of daylight.



Sketching Other Graphs 1

1. What is the mathematical name for the graph of $y = \frac{1}{x}$?

2. What are the maximum and minimum values for the graph $y = cos\theta$?

3. Sketch the graph of $y = 2^x$.
   Label the y and x intercepts.

4. Using a sketch of the graphs
   $$y = \frac{1}{x} \text{ and } y = x$$
   Show how many solutions there will be to the equation $\frac{1}{x} = x$

5. What is the name for this type of graph?

6. What is the y intercept of the graph
   $y = (x + 2)(x - 3)(x + 5)$?

7. What are the x intercepts of the graph
   $y = (x + 2)(x - 3)(x + 5)$?

8. Sketch the graph of
   $y = (x - 3)(x + 2)(x + 5)$

Sketching Other Graphs 2

1. What is the mathematical name for graphs of the form of $x^2 + y^2 = 9$?

2. Sketch the graph of $y = sin\theta$ between $0°$ and $360°$, labelling x and y intercepts

3. On your sketch for Q2 draw in the line
   $$y = 0.5$$
   How many solutions are there to
   $$sin\theta = 0.5?$$
   Can you say what they are?

4. Sketch the graph $y = x^3$, labelling any intersections

5. Sketch the graph of the equation in Q1, label any intersections with the x and y axis

6. What is the y intercept of the graph
   $y = (x + 1)(x + 1)(x - 1)$?

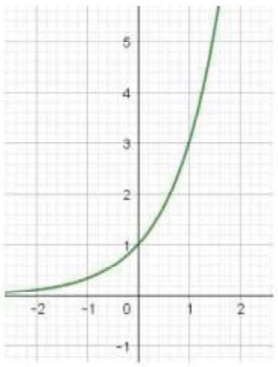
7. What are the x intercepts of the graph
   $y = (x + 1)(x + 1)(x - 1)$?

8. Sketch the graphs of
   $$x^2 + y^2 = 4$$
   $$y = x + 1$$
   Use the sketch to determine how many solutions there are when those equations are solved simultaneously

Which is which?

Match the graphs to the equations - there are more equations than you need!

A



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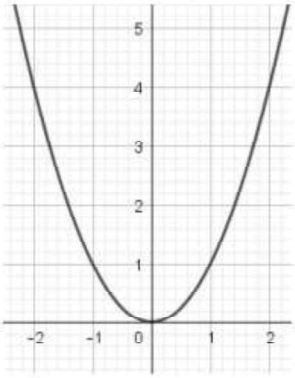

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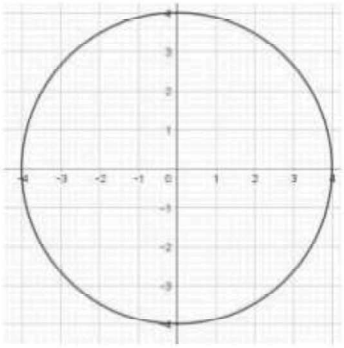

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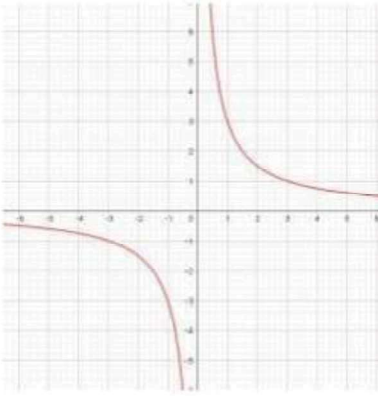

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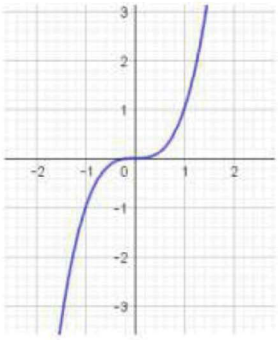

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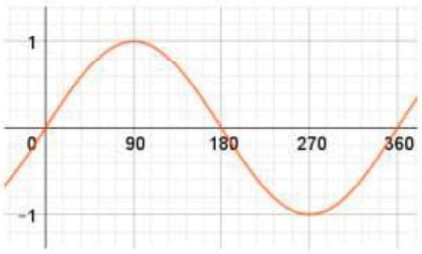


- $y = 3^x$
- $y = \frac{3}{x}$
- $y = sin\theta$
- $y = 2x^2$
- $y = tan\theta$
- $y^2 + x^2 = 16$
- $y = x^2$
- $y = x^3$
- $y = \frac{1}{x}$
- $y = x^2 + 3$

Shortest Distance

Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$
$$y = x^2 + 7$$

How fast?

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min.

It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

- What is the car's average speed for the first 20 seconds of travel?
- What is the car's deceleration?

A square in a circle.

A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.

Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?

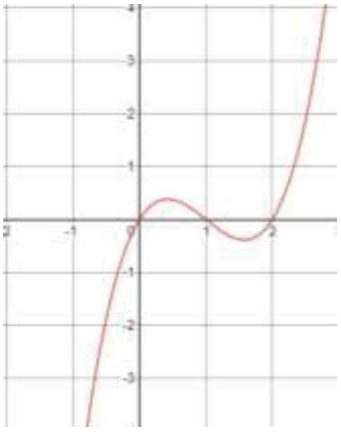
Hint: Assume C_2 has a radius of 1 unit

A Triggy Problem!

Solve $(\sin x + 1)(2\cos x - 1) = 0$ for $0 < x < 360°$

A cubic match up

Which one of the equations below describes the graph?



1. $y = (x + 1)(x - 1)(x - 2)$
2. $y = -x(x - 1)(x + 1)$
3. $y = x(x - 1)(x + 1)$

Catching Stars

Go to Student.Desmos.com (use classroom code: **FENFZP**) to try an Exponential Marbleslides Challenge.

You will be investigating the features of exponential graphs whilst trying to catch as many stars as possible.

You can join the activity without signing in or entering your real name.

Solving Other Equations

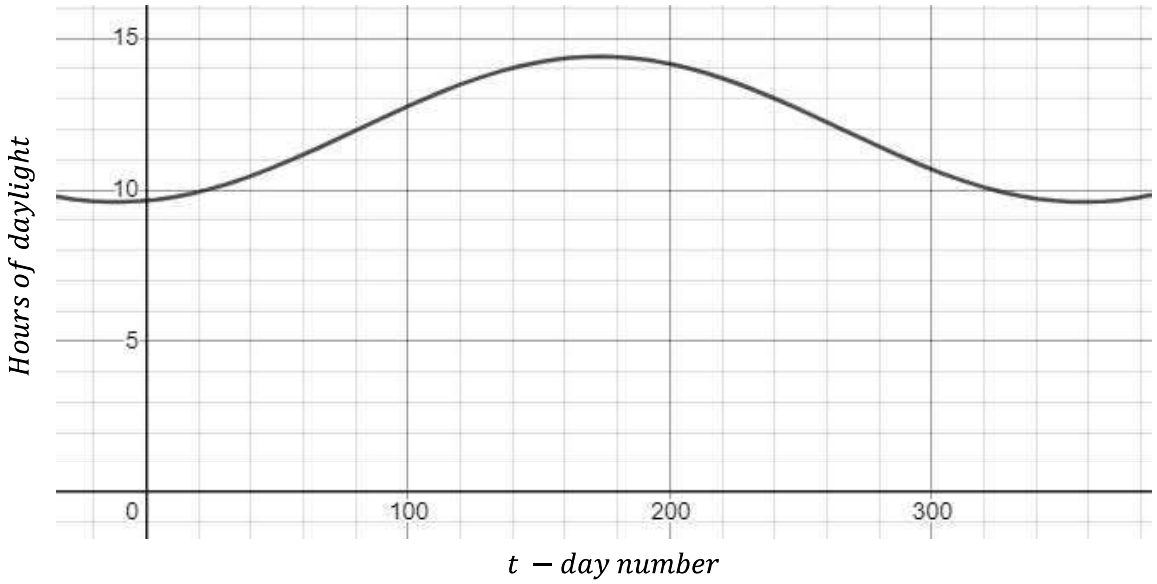
Did you know?

Sunrise and sunset times are modelled using trigonometrical equations

For San Diego, California, a simple equation to model daylight hours would be:

$$\text{Number of daylight hours} = 2.4\sin(0.017t - 1.377) + 12$$

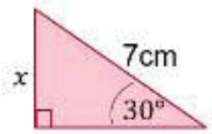
where t is the day of year from 0 to 365



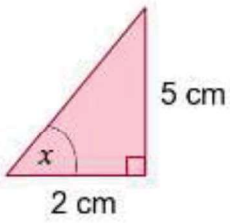
- From the graph can you tell which dates of the year are the shortest and longest day?

Solving Equations with Trigonometry

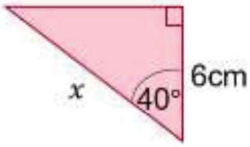
1. Calculate the length of the side marked x in this triangle.



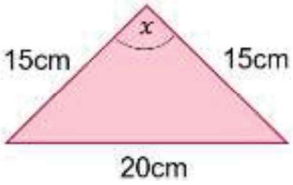
2. Calculate the value of the angle marked x in this triangle.



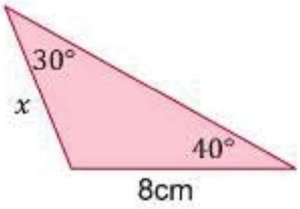
3. Calculate the value of the side marked x in this triangle



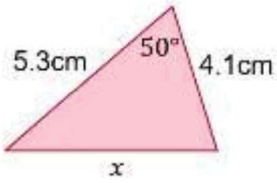
4. Calculate the value of the angle marked x in this triangle.



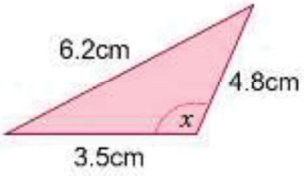
5. Calculate the value of the side marked x in this triangle



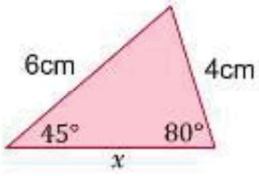
6. Calculate the value of the side marked x in this triangle.



7. Calculate the value of the angle marked x in this triangle.



8. Calculate the value of the side marked x in this triangle.

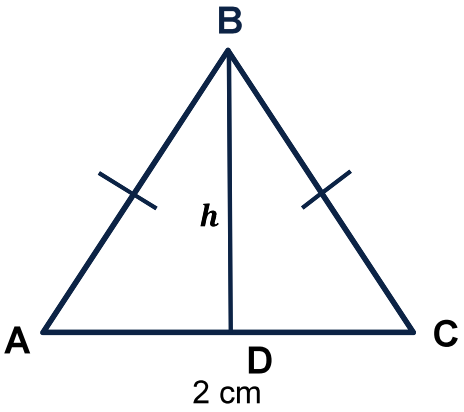


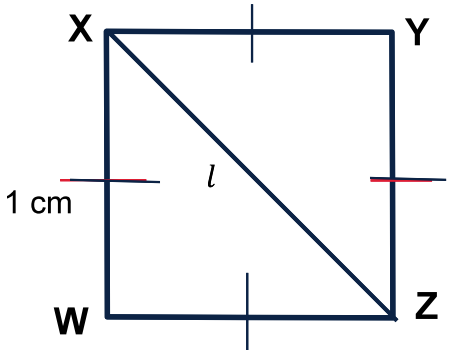
Other Equations

Solve the following

1. $3^x = 243$
2. $2^{2x+3} = 128$ **Hint:** write 128 as powers of 2
3. $\sqrt{x+3} = 7$
4. $2\sqrt{x} + 1 = \sqrt{12} + 3$
5. $3\sqrt{x} + 12 = 7\sqrt{x}$
6. $\sin x = \frac{1}{2} \quad 0 \leq x \leq 360$
7. $\cos x = 0.866 \quad 0 \leq x \leq 360$
8. $\frac{8}{3x+7} = 2$

Missing info





	Answer
Length of AB	
Length of BD	
Length of AD	
Size of $\angle BAD$	
Size of $\angle ABD$	

	Answer
Length of WZ	
Length of XZ	
Size of $\angle WZX$	
Size of $\angle WXZ$	

Use your knowledge of regular shapes to complete the tables above (you will need them for the next task).

Let's get Triggy

Use your tables and diagrams from the previous activity to complete this table

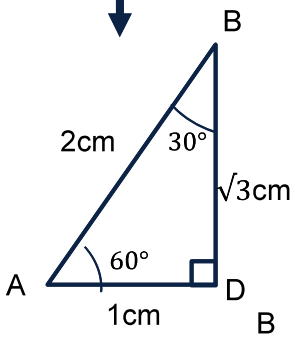
θ	$30°$	$45°$	$60°$
$sin\theta$	$\frac{}{AB} = \frac{1}{2}$	$\frac{XW}{} = \frac{WZ}{XZ} = \frac{}{}$	$\frac{}{AB} = \frac{}{}$
$cos\theta$	$\frac{}{} = \frac{\sqrt{3}}{}$	$\frac{}{} = \frac{WZ}{} = \frac{}{}$	$\frac{}{} = \frac{}{}$
$tan\theta$	$\frac{}{} = \frac{}{\sqrt{3}}$	$\frac{}{} = \frac{}{} = 1$	$\frac{}{} = \frac{}{1} = \sqrt{}$

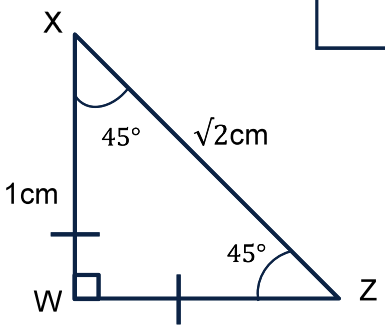
Let's get Triggy Hint

Use your tables and diagrams from the previous activity to complete this table

Some examples are filled in to get you started

These will help





θ	$30°$	$45°$	$60°$
$sin\theta$ =	$\frac{AD}{AB} = \frac{1}{2}$	$\frac{XW}{\quad} = \frac{WZ}{XZ} = \frac{\quad}{\quad}$	$\frac{\quad}{AB} = \frac{\sqrt{3}}{\quad}$
$cos\theta$ =	$\frac{\quad}{\quad} = \frac{\sqrt{3}}{\quad}$	$\frac{\quad}{\quad} = \frac{WZ}{XW} = \frac{\quad}{\quad}$	$\frac{AD}{AB} = \frac{\quad}{\quad}$
$tan\theta$ =	$\frac{\quad}{\quad} = \frac{\quad}{\sqrt{3}}$	$\frac{\quad}{\quad} = \frac{\quad}{\quad} = 1$	$\frac{\quad}{\quad} = \frac{\quad}{1} = \sqrt{\quad}$

Trig Maze

Starting at $\sqrt{3}$ on the left hand side of the rectangle, find your way to the right hand side by landing only on expressions that are equivalent to $\sqrt{3}$

$\frac{\tan 30°}{3}$	$\frac{9}{3^{0.5}}$	$\frac{\sqrt{18}}{\sqrt{6}}$	$\frac{1.5}{0.05}$	$\frac{\sqrt{12}}{\sqrt{2}}$	$\frac{2\sqrt{6}}{\sqrt{4}}$	$\frac{\sqrt{9}}{3^0}$
$\frac{\sqrt{27}}{3}$	$\frac{3\sqrt{3}}{\sqrt{3}}$	$2\cos 60°$	$\frac{\tan 60°}{2}$	$\frac{\sin 30°}{\cos 30°}$	$3\tan 30°$	$\frac{\sqrt{6}}{\sqrt{2}}$
$\frac{6}{\sqrt{2}}$	$\frac{\cos 60°}{\sin 60°}$	$\frac{9}{3\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$2\cos 30°$	$\frac{3+\sqrt{3}}{\sqrt{3}}$ -1	$3\tan 60°$
$\sqrt{3}$	$\frac{9}{\sqrt{3}}$	$2\sin 60°$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{9}}{\sqrt{3}}$	$\frac{\sqrt{6}}{2}$	$\frac{\cos 30°}{2}$
$3^{\frac{1}{2}}$	$\tan 60°$	$\frac{\sqrt{12}}{2}$	$2\sin 30°$	$\frac{\sin 60°}{\cos 60°}$	$\frac{9^{0.5}}{3^{0.5}}$	$\frac{2\sqrt{6}}{\sqrt{8}}$
$\frac{\cos 60°}{2}$	$\frac{\sqrt{12}}{4}$	$\frac{\sin 30°}{2}$	$\frac{\sqrt{9}}{3}$	$\frac{\tan 60°}{3}$	$\frac{9 \times 10^1}{3 \times 10^{-1}}$	$\frac{3+\sqrt{3}}{\sqrt{3}}$

Triggy Problems

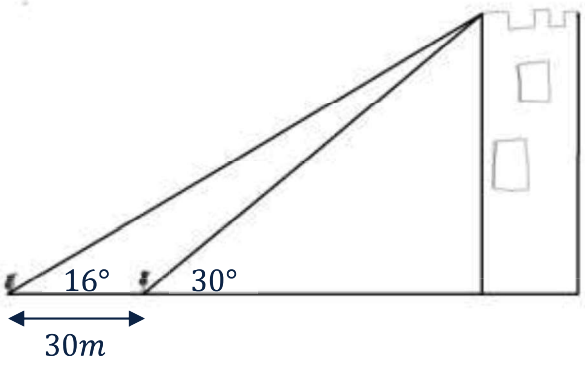
1. The area of an equilateral triangle is $10\ cm^2$.

   What are the lengths of the sides?

2. Two birds are sitting looking at the top of a tower block, as shown in the diagram

   They are 30m apart.

   How tall is the tower?



Multiple Equations

If $\frac{ab}{a+b} = \frac{1}{4}$ and $\frac{bc}{b+c} = \frac{1}{2}$ and $\frac{ac}{a+c} = \frac{1}{8}$ find a, b and c

Hint:

- Rearrange these equations so they are linear i.e. no fractions
- Find an expression for b and c in terms of a
- Substitute into the equation that uses b and c

Powers

Using what you know about powers, can you solve this equation

$$(x-6)^{x^2-9}=1$$

Hint

- What do you know about a^0
- What do you know about 1^a
- What do you know about $(-1)^a$

Geometry Puzzle